An Achievable Scheme for the K-user Linear Computation Broadcast Channel


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Outline

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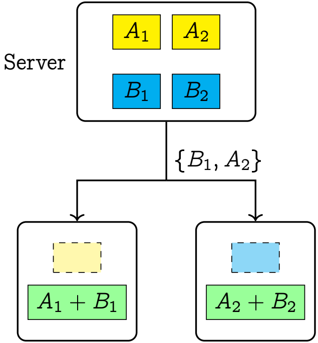
What is K-LCBC?

- A K-LCBC comprises data stored on a server and K users.
- Each user aims to retrieve a desired linear function of data.
- Users leverage their prior side information (also a linear function of data) to retrive their desired information.
- The server broadcasts a message to the users so as to minimize the communication load.



Motivation & Contributions

- Motivated by distributed computing (e.g., federated learning), and related to other models in info. theory.
- Index coding was instrumental to determine the minimal load of the LCBC with uncoded placement.
- LCBC could potentially be leveraged to derive the ultimate performance limits of coded caching with linear coded placement.

Motivation & Contributions

- Motivated by distributed computing (e.g., federated learning), and related to other models in info. theory.
- Index coding was instrumental to determine the minimal load of the LCBC with uncoded placement.
- LCBC could potentially be leveraged to derive the ultimate performance limits of coded caching with linear coded placement.
- Prior LCBC works include optimality results for 2 and 3 users, and a 'generic' setting with any number of users.
- We introduce an LP-framework for K-LCBC achievability by multicast and interference elimination.

System Model

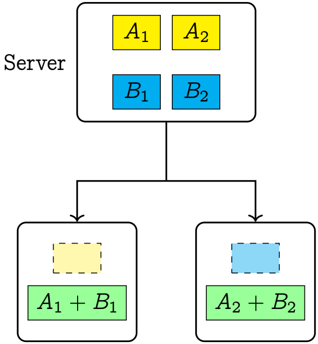
- Server stores data i.i.d. uniformly at random

$$\mathrm{f}(t) = [f_1(t), \ldots, f_{\mathsf{d}}(t)]^T \in \mathbb{F}_{\mathsf{q}}^{\mathsf{d}\times 1}.$$

- Side info. for user k: $\mathrm{w}'_k(t) = \mathrm{f}(t)^T \mathrm{V}'_k \in \mathbb{F}_{\mathsf{q}}^{1\times m'_k}$.
- Desired info. for user k: $\mathrm{w}_k(t) = \mathrm{f}(t)^T \mathrm{V}_k \in \mathbb{F}_{\mathsf{q}}^{1\times m_k}$.
- WLOG, V'_k and V_k are linearly independent.

System Model (Cont.)

$$\begin{aligned}
\mathrm{d} &= 4, m'_\star = 1, m_\star = 2,\\
\mathrm{f}(t) &= [A_1, A_2, B_1, B_2]^T,\\
\mathrm{V}'_1 &= [1, 0, 1, 0],\\
\mathrm{V}'_2 &= [0, 1, 0, 1],\\
\mathrm{V}_1 &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix},\\
\mathrm{V}_2 &= \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.
\end{aligned}$$



Performance Metric

An achievable $(L, N, \phi, (\varphi_k)_{k\in[\mathsf{K}]})$ coding scheme consists of:

- Server aggregates L instances: $\mathrm{F} = [\mathrm{f}(1), \ldots, \mathrm{f}(L)]$.
- Server sends $X = \phi(\mathrm{F})$ of size N.
- User k decodes $\mathrm{W}_k = \varphi_k(\mathrm{W}'_k, X)$.
- Rate: $R = N/L$.
- LCBC Capacity: The infimum of N/L over all achievable schemes.
- Capacity for 2 and 3 users is known. Capacity for $\mathsf{K} \geq 4$ users is open.

General Subspace Decomposition

- Our proposed scheme is based on a subspace decomposition derived from representable polymatroid spaces.
- This enables the server to design multicast messages that simultaneously benefit multiple users.
- Users can eliminate interference using their available side information.
- Notation:
  - Matrix: M. If columns are linear indept., they are a base.
  - Space spanned by columns of M: $\langle \mathrm{M} \rangle$.
  - Intersection of subspaces: $\mathrm{M}_{\mathcal{S}} := \bigcap_{k \in \mathcal{S}} \langle \mathrm{M}_k \rangle$.
  - Union of subspaces: $\mathrm{M}_{(\mathcal{S})} := \bigcup_{k \in \mathcal{S}} \langle \mathrm{M}_k \rangle$.

General Subspace Decomposition (Cont.)

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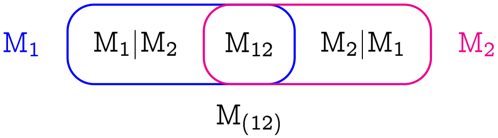
- Rank Notation:
  - Conditional Rank:

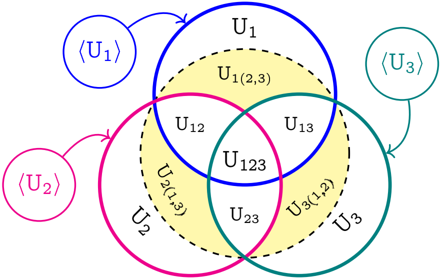
$$\mathsf{rk}(\mathrm{M}_1 | \mathrm{M}_2) := \mathsf{rk}(\mathrm{M}_{(12)}) - \mathsf{rk}(\mathrm{M}_2) = \mathsf{rk}(\mathrm{M}_1) - \mathsf{rk}(\mathrm{M}_{12}).$$



Subspace & Basis

Given $\{\mathrm{U}_k : k \in [\mathsf{K}]\}$, define $\mathcal{V} := \mathcal{V}_1 \cup \mathcal{V}_2 \cup \mathcal{V}_3$:

- $\mathcal{V}_1 = \{\mathrm{U}_{\mathcal{S}} : \mathcal{S} \subseteq [\mathsf{K}], |\mathcal{S}| \geq 2\}$.
- $\mathcal{V}_2 = \{\mathrm{U}_{k(\mathcal{T} \setminus k)} : \mathcal{T} \subseteq [\mathsf{K}], |\mathcal{T}| \geq 3, k \in \mathcal{T}\}$.
- $\mathcal{V}_3 = \{\mathrm{U}_k, k \in [\mathsf{K}]\}$.

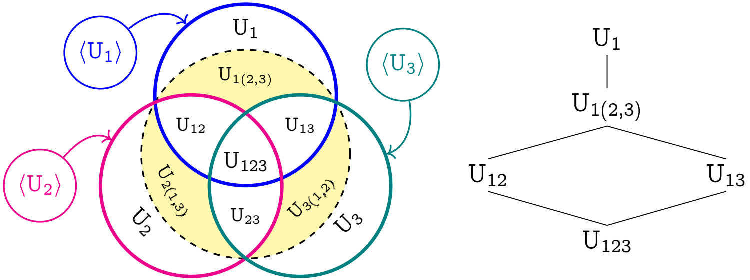


$$\mathcal{V}_1 = \{\mathrm{U}_{123}, \mathrm{U}_{12}, \mathrm{U}_{13}, \mathrm{U}_{23}\},$$
$$\mathcal{V}_2 = \{\mathrm{U}_{1(2,3)}, \mathrm{U}_{2(1,3)}, \mathrm{U}_{3(1,2)}\},$$
$$\mathcal{V}_3 = \{\mathrm{U}_1, \mathrm{U}_2, \mathrm{U}_3\}.$$

Subspace & Basis (Cont.)

Given $\{\mathrm{U}_k : k \in [\mathsf{K}]\}$, define $\mathcal{V} := \mathcal{V}_1 \cup \mathcal{V}_2 \cup \mathcal{V}_3$:

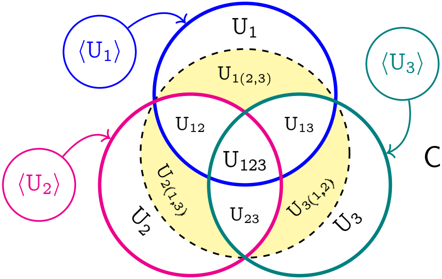
- For a subspace $V \in \mathcal{V}$, $\mathsf{LS}(V)$ lists the "closest" subspaces contained in V.
- An ordered tree is possible by connecting node V and every node in $\mathsf{LS}(V)$.



Subspace & Basis (Cont.)

Given $\{\mathsf{U}_k : k \in [\mathsf{K}]\}$, define $\mathcal{V} := \mathcal{V}_1 \cup \mathcal{V}_2 \cup \mathcal{V}_3$:

- Basis $\mathsf{B}(V) := V \setminus \bigcup_{Q \in \mathsf{LS}(V)} Q$. This spans V but not the subspaces in $\mathsf{LS}(V)$.
- $\mathsf{C}(V) = \mathsf{B}(V) \cup \left(\bigcup_{U \in \mathsf{LS}(V)} \mathsf{C}(U)\right)$ is the set of bases covered by V.

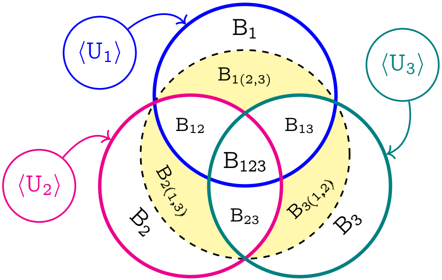


$$\mathsf{C}(\mathrm{U}_{12}) = \{\mathrm{B}_{12}, \mathrm{B}_{123}\},$$
$$\mathsf{C}(\mathrm{U}_{1(2,3)}) = \{\mathrm{B}_{(123)}, \mathrm{B}_{12}, \mathrm{B}_{13}, \mathrm{B}_{123}\},$$
$$\mathsf{C}(\mathrm{U}_{1}) = \{\mathrm{B}_{1}, \mathrm{B}_{(123)}, \mathrm{B}_{12}, \mathrm{B}_{13}, \mathrm{B}_{123}\}.$$

Subspace & Basis (Cont.)

Lemma

Given any $\mathcal{T} \subseteq [\mathsf{K}]$ and $t := |\mathcal{T}| \geq 3$, the subspaces $\mathrm{B}_{k(\mathcal{T}\setminus\{k\})} : k \in \mathcal{T}$, have identical dimension, and any $t-1$ of them are linearly independent.

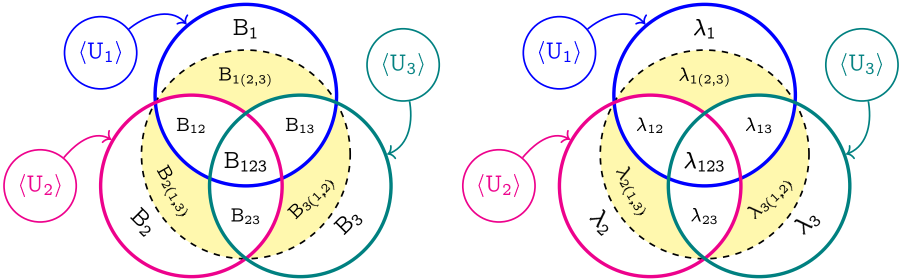


When $\mathsf{K} = 3$, [YJ24a] shows $\mathrm{B}_{1(2,3)}$, $\mathrm{B}_{2(1,3)}$ and $\mathrm{B}_{3(1,2)}$ are pairwise linearly independent, and have same dimension.

Theorem 1: Achievability via LP

Let $\lambda_\star \geq 0$ be the optimizing variables, essentially coding gain from linear coding.

- For every $\mathrm{U}_\mathcal{S} \in \mathcal{V}_1$, $\lambda_\mathcal{S}$ is assigned to its base $\mathrm{B}_\mathcal{S}$;
- For every $\mathrm{U}_{k(\mathcal{S})} \in \mathcal{V}_2$, $\lambda_{(\{k\}\cup\mathcal{S})}$ is assigned to $\mathrm{B}_{k(\mathcal{S})}$;
- For every U_k, λ_k is assigned to B_k.



Theorem 1: Achievability via LP

For the K-LCBC, let $U_k = [V'_k; V_k]$. The following rate is achievable:

$$\min_{\lambda_\star \geq 0} \sum_{\mathcal{S} \subseteq [K]} \lambda_{\mathcal{S}} + \sum_{t=3}^{K} \sum_{\mathcal{S} \in \Omega_{[K]}^{t}} (t-1)\lambda_{(\mathcal{S})}$$

Where $\lambda_\star$ are varibales associated with bases from $\mathcal{V}_1, \mathcal{V}_2, \mathcal{V}_3$.

- For $U_{\mathcal{S}} \in \mathcal{V}_1$ and $\mathcal{L}_{\mathcal{S}} \subseteq \mathsf{LC}(U_{\mathcal{S}})$,

$$\text{"Sum of } \mathsf{C}_\lambda(U_{\mathcal{S}})\text{"} \leq \mathsf{rk}(U_{\mathcal{S}} \mid V'_k), \tag{1a}$$

$$\text{"Sum of } \bigcup_{V \in \mathcal{L}_{\mathcal{S}}} \mathsf{C}_\lambda(V)\text{"} \leq \mathsf{rk}(\mathcal{L}_{\mathcal{S}} \mid V'_k), \tag{1b}$$

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- For $\mathrm{U}_{\mathcal{S}} \in \mathcal{V}_1$ and $\mathcal{L}_{\mathcal{S}} \subseteq \mathsf{LC}(\mathrm{U}_{\mathcal{S}})$,

$$\text{"Sum of } \mathsf{C}_\lambda(\mathrm{U}_{\mathcal{S}})\text{"} \leq \mathsf{rk}(\mathrm{U}_{\mathcal{S}} \mid \mathrm{V}'_k), \tag{1a}$$

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- For $\mathrm{U}_{k(\mathcal{S})} \in \mathcal{V}_2$ and $\mathcal{L}_{k(\mathcal{S})} \subseteq \mathsf{LC}(\mathrm{U}_{k(\mathcal{S})})$,

$$\text{"Sum of } \mathsf{C}_\lambda(\mathrm{U}_{k(\mathcal{S})})\text{"} \leq \mathsf{rk}(\mathrm{U}_{k(\mathcal{S})} \mid \mathrm{V}'_k), \tag{2a}$$

$$\text{"Sum of } \bigcup_{V \in \mathcal{L}_{k(\mathcal{S})}} \mathsf{C}_\lambda(V)\text{"} \leq \mathsf{rk}(\mathcal{L}_{k(\mathcal{S})} \mid \mathrm{V}'_k), \tag{2b}$$

Theorem 1: Achievable Rate via LP

For the K-LCBC, let $U_k = [V'_k; V_k]$. The following rate is achievable:

$$\min_{\lambda_\star \geq 0} \sum_{\mathcal{S} \subseteq [K]} \lambda_{\mathcal{S}} + \sum_{t=3}^{K} \sum_{\mathcal{S} \in \Omega^t_{[K]}} (t-1)\lambda_{(\mathcal{S})}$$

Where $\lambda_\star$ are coefficients associated with bases from $\mathcal{V}_1, \mathcal{V}_2, \mathcal{V}_3$.

- ...
- For $U_k \in \mathcal{V}_3$,

$$\text{"Sum of } C_\lambda(U_k)\text{"} = \mathrm{rk}(U_k \mid V'_k) = \mathrm{rk}(V_k), \qquad (3)$$

- Equality ensures all desired information is recovered.
- When $K = 2$ or $K = 3$, this LP is proved as optimal in [SJ19] and [YJ24a], respectively.

Interpreting λ Variables & Coding Gain

The objective function can be rewritten as:

$$\sum_{k=1}^{\mathsf{K}} \mathrm{rk}(\mathsf{U}_k|\mathsf{V}'_k) - \sum_{t=2}^{\mathsf{K}} \sum_{\mathcal{S}\in\Omega^t_{[\mathsf{K}]}} (t-1)\lambda_{\mathcal{S}} - \sum_{\mathcal{S}\subseteq[\mathsf{K}],|\mathcal{S}|\geq 3} \lambda_{(\mathcal{S})}$$

- First term: Load of uncoded transmissions (serving users individually).
- Each $\lambda_\star$ represents the coding gain from a multicast message.

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- First term: Load of uncoded transmissions (serving users individually).
- Each $\lambda_\star$ represents the coding gain from a multicast message.
- $\lambda_{\mathcal{S}}$: Rank of (multicast or unicast when $|\mathcal{S}|=1$) message serving $|\mathcal{S}|$ users, reducing load by $(|\mathcal{S}|-1)\lambda_{\mathcal{S}}$.
- $\lambda_{(\mathcal{S})}$: Rank of messages where $t-1$ are linearly independent, serving $t=|\mathcal{S}|$ users, reducing load by $\lambda_{(\mathcal{S})}$.

Example: K=4 Users

Toy Example (Index Coding):

- $\mathrm{f} = [A, B, C, D]$.
- User 1 wants B, knows A; User 2 wants C, knows B; User 3 wants D, knows C; User 4 wants A, knows D.
- Subspace decomposition
  - $\mathrm{U}_1 = [A, B], \mathrm{U}_2 = [B, C], \mathrm{U}_3 = [C, D], \mathrm{U}_4 = [A, D]$.
  - $\mathrm{B}_{12} = B, \quad \mathrm{B}_{23} = C, \quad \mathrm{B}_{34} = D, \quad \mathrm{B}_{14} = A.$
- Objective function

$$\sum_{\mathcal{S}\subseteq[4]} \lambda_{\mathcal{S}} + \sum_{t=3}^{4} \sum_{\mathcal{S}\in\Omega_{[4]}^{t}} (t-1)\lambda_{(\mathcal{S})}.$$

- Constraints on $\lambda_\star$
  - $\lambda_{12} \leq \mathsf{rk}(\mathrm{U}_{12}|\mathrm{V}_1') = \mathsf{rk}([A, B]) - \mathsf{rk}([B]) = 1,$
  - $\lambda_{12} \leq \mathsf{rk}(\mathrm{U}_{12}|\mathrm{V}_2') = \mathsf{rk}([B, B]) - \mathsf{rk}([B]) = 0$
  - All $\lambda_{\mathcal{S}} = 0$ where $\mathcal{S} \subseteq [4], |\mathcal{S}| \geq 2.$

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- Objective function

$$\sum_{\mathcal{S} \subseteq [4]} \lambda_{\mathcal{S}} + \sum_{t=3}^{4} \sum_{\mathcal{S} \in \Omega_{[4]}^{t}} (t-1)\lambda_{(\mathcal{S})}.$$

- Constraints on $\lambda_{\star}$
  - $\lambda_{(123)} \leq \mathsf{rk}(\mathrm{U}_{1(23)}|\mathrm{V}_1') = \mathsf{rk}([A]) - \mathsf{rk}([A]) = 0,$
  - $\lambda_{(123)} \leq \mathsf{rk}(\mathrm{U}_{2(13)}|\mathrm{V}_2') = \mathsf{rk}([A, B]) - \mathsf{rk}([B]) = 1,$
  - $\lambda_{(123)} \leq \mathsf{rk}(\mathrm{U}_{3(12)}|\mathrm{V}_3') = \mathsf{rk}([C, C]) - \mathsf{rk}([C]) = 0,$
  - $\lambda_{(123)} = \lambda_{(124)} = \lambda_{(134)} = \lambda_{(234)} = 0.$

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  - $\mathrm{U}_1 = [A, B], \mathrm{U}_2 = [B, C], \mathrm{U}_3 = [C, D], \mathrm{U}_4 = [A, D]$.
  - $\mathrm{B}_{12} = B, \quad \mathrm{B}_{23} = C, \quad \mathrm{B}_{34} = D, \quad \mathrm{B}_{14} = A.$
- Objective function

$$\sum_{\mathcal{S}\subseteq[4]} \lambda_{\mathcal{S}} + \sum_{t=3}^{4} \sum_{\mathcal{S}\in\Omega^t_{[4]}} (t-1)\lambda_{(\mathcal{S})}.$$

- Constraints on $\lambda_\star$
  - $\lambda_{(1234)} \leq \mathsf{rk}(\mathrm{U}_{1(234)}|\mathrm{V}_1') = \mathsf{rk}([A, B]) - \mathsf{rk}([A]) = 1,$
  - $\lambda_{(1234)} \leq \mathsf{rk}(\mathrm{U}_{2(134)}|\mathrm{V}_2') = \mathsf{rk}([B, C]) - \mathsf{rk}([B]) = 1,$
  - $\lambda_{(1234)} \leq \mathsf{rk}(\mathrm{U}_{3(124)}|\mathrm{V}_3') = \mathsf{rk}([C, D]) - \mathsf{rk}([C]) = 1,$
  - $\lambda_{(1234)} \leq \mathsf{rk}(\mathrm{U}_{4(123)}|\mathrm{V}_4') = \mathsf{rk}([A, D]) - \mathsf{rk}([D]) = 1,$

Example: K=4 Users

Toy Example (Index Coding):

- $\mathrm{f} = [A, B, C, D]$.
- User 1 wants B, knows A; User 2 wants C, knows B; User 3 wants D, knows C; User 4 wants A, knows D.
- Subspace decomposition
  - $\mathrm{U}_1 = [A, B], \mathrm{U}_2 = [B, C], \mathrm{U}_3 = [C, D], \mathrm{U}_4 = [A, D]$.
  - $\mathrm{B}_{12} = B, \quad \mathrm{B}_{23} = C, \quad \mathrm{B}_{34} = D, \quad \mathrm{B}_{14} = A.$
- Objective function

$$\sum_{\mathcal{S} \subseteq [4]} \lambda_{\mathcal{S}} + \sum_{t=3}^{4} \sum_{\mathcal{S} \in \Omega_{[4]}^{t}} (t-1)\lambda_{(\mathcal{S})}.$$

- Constraints on $\lambda_\star$: $\lambda_{(1234)} \leq 1, \lambda_k \leq 1$,
- "Four birds Three stones", $X = (A+B, B+C, C+D)$.
- Optimal as proved by [AK+18, MAIS converse].

Generic LCBC Setting

- Consider a LCBC instance Λ_n where
  - uniform cache size and demand size;
  - V_k' and V_k are generated from $\mathbb{F}_{p^n}$ at i.i.d. uniformly random;
- Performance Metric
  - Find Δ s.t. $\Pr(|\Delta^\star(\Lambda_n) - \Delta| < \epsilon) \to 1$ when $n \to \infty$.
  - Since $\Delta^\star(\Lambda_n)$ is always bounded, it implies $\mathbb{E}\Delta^\star(\Lambda_n) = \Delta$.
- [YJ24b] proves such exact $\Delta^\star$ exists when certain conditions hold, if not, it is within a factor of 2.
  - The scheme invokes asymptotic IA scheme in K-user interference channel.

Example: K=4 Users

Example (Generic LCBC setting)

- f $= [A, B, C, D]$, d $= 4$, $m'_k = m_k = 1$.
- $\mathrm{V}'_k = [v'_{k,i} : i \in [4]]$, $\mathrm{V}_k = [v_{k,i} : i \in [4]]$.
- $[\mathrm{V}'_1; \ldots; \mathrm{V}'_4; \mathrm{V}_1; \ldots \mathrm{V}_4]$ is $(4, 8)$ MDS code.

Subspace decomposition ($ijr\ell$ is a permutation of 1234)

- $\mathrm{B}_{i(jr)} = \mathrm{U}_i$, other bases are 0.

Constraints on $\lambda_\star$

$$\lambda_{(ijr)} \leq \mathrm{rk}(\mathrm{U}_{i(jr)} | \mathrm{V}'_i) = 1, \tag{4a}$$

$$\lambda_{(1234)} \leq \mathrm{rk}(\mathrm{U}_{i(jr\ell)} | \mathrm{V}'_i) = 1 \tag{4b}$$

$$\lambda_{(ijr)} + \lambda_{(ij\ell)} \leq \mathrm{rk}(\mathrm{U}_{i(jr)}, \mathrm{U}_{i(j\ell)} | \mathrm{V}'_i) = 1, \tag{4c}$$

$$\lambda_{(ijr)} + \lambda_{(ij\ell)} + \lambda_{(ir\ell)} \leq \mathrm{rk}(\mathrm{U}_{i(jr)}, \mathrm{U}_{i(j\ell)}, \mathrm{U}_{i(r\ell)} | \mathrm{V}'_i) = 1, \tag{4d}$$

Solving LP, $\lambda_{(ijr)} = \frac{1}{3}$, thus the load is $\frac{8}{3}$. However, [YJ24b] proves the exact load is 2.

Conclusion

- We propose a new achievable scheme for general K-LCBC (i.e., arbitrary number of users K).
- The scheme is based on a general subspace decomposition derived from representable polymatroid spaces.
- It leverages linear dependencies among subspaces to enable multicast opportunities and interference elimination, optimizing the communication load via a linear program.
- The scheme recovers known capacity results for $\mathsf{K} = 2$ and $\mathsf{K} = 3$ users.

Future Work

- Comparing the achievable rate with a converse bound to determine optimality or sub-optimality gaps.
- Further strengthening the scheme by accounting for more complex dependencies that may exist in the subspace decomposition, potentially lowering the rate.
- Exploring applications to related problems, such as:
  - Coded caching with linear coded placement.
  - Coded caching with scalar linear function retrieval.

Thank You!

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